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A SINGULAR PERTURBATIONS APPROACH TO REDUCED-ORDER MODELING AND DECOUPLING FOR LARGE SCALE LINEAR SYSTEMS

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Singular perturbation concepts have been very helpful in control and systems theory, both in eliminating difficulties associated with stiffness and in reducing dimensionality (cf. Kokotovic et al. (1976) and O'Malley (1978)). In this short report, we wish to emphasize the second aspect, especially in the context of large-scale problems. Such an approach has been applied to power system modeling (cf. Kokotovic et al. (1979)) and to aircraft maneuvering and structural dynamics (cf. Anderson (1978) and Anderson and Hallauer (1980)).

Let us first consider initial value problems on a bounded interval, $0 \le t \le T$, for linear systems of the form

$$\dot{x} = Ax + Bu , \tag{1}$$

where A and B are constant matrices and the control $u(t)$ is considered, for simplicity in the present discussion, to be given.

A low-order example is provided by a model of the longitudinal dynamics of an F-8 aircraft (cf. Etkin (1972) and Teneketzis and Sandell (1977)). It involves four states x_i and one control u, the elevator deflection. Two of the four states, velocity variation and flight path angle, are described through physical intuition as being "primarily (or predominantly) slow" variables compared to the other "primarily fast" states, angle of attack and pitch rate. It is a natural temptation in such a situation to seek to approximate the solution away from $t = 0$ by neglecting the dynamics of the primarily fast variables and to integrate only the resulting initial value problem for the slow states. (This has often been proposed in the engineering literature, cf., e.g., Harvey and Pope (1976)). The trouble with this approach is that the physical coordinates are not actually provided separately as slow and fast modes, in contrast to the situation for the traditional singularly perturbed system $\dot{\alpha} = f(\alpha,\beta,t)$, $\varepsilon\dot{\beta} = g(\alpha,\beta,t)$ where β can respond on a much faster time-scale than α as $\varepsilon \to 0$.

The analytical solution of the linear system (1) is commonly expressed using variation of parameters and the matrix exponential solution e^{At} of the homogeneous system. Moler and van Loan's 1978 article "Nineteen dubious ways to compute the exponential of a matrix" illustrates the difficulties involved in computationally exploiting specific representations for this fundamental matrix. The difficulties are compounded when the dimensions of A are large. We, nonetheless, recall that the columns of e^{At} are of the form $v_i(t)e^{\lambda_i t}$ where v_i is a vector polynomial in t in the span of the eigenspace corresponding to the eigenvalue λ_i of A. We shall seek an approximation away from $t = 0$ which avoids doing a complete eigenanalysis of A. Our approximation will involve neglecting modes corresponding to large stable eigenvalues of A. This corresponds to the reduced order model in singular perturbations (cf. O'Malley (1974)) and to the smooth approximate solution of the stiff equations literature (cf. Dahlquist (1969) and Oden (1971)).

For the F-8 aircraft model, we have the two "slow" (i.e., small in magnitude) eigenvalues $s_{1,2} = (-0.75 \pm i7.6) \times 10^{-2}$ compared to the two "fast" (i.e., large) eigenvalues $f_{1,2} = -0.94 \pm i3.0$. The smallness of the parameter $\mu = |s_2/f_1| = 0.024$ indicates a large time-scale separation between slow and fast modes, and the smallness of $\varepsilon = -(1/\mathrm{Re}\, f_1)/T = 1.06/T$ for a long enough time interval $0 \leq t \leq T$ provides fast-mode stability and a rapid transition to pseudo-steady state behavior. We shall determine a second-order nonstiff model for the solution away from $t = 0$. A basic question is what initial values should be used for the second-order problem. (The need to "project" the initial vector occurs in many related situations (cf. O'Malley and Flaherty (1980), Lentini and Keller (1980), and de Hoog and Weiss (1979))). Our approximation, based on asymptotic methods, will improve as the parameters ε and μ both tend toward zero. In practice, however, there is often a need to use such reduced-order methods even when these parameters are only moderately small. In contrast, our related suggestion of "equilibrating" the primarily fast states will produce an error which persists for all $t > 0$ (cf. Figure 1).

In the applications of interest, very high dimensional systems are common. We shall seek approximate nonstiff models of (differential) order $n_1 << n$, the dimension of x, where n_1 is the number of "slow", or small in magnitude, eigenvalues of A compared to its other (large) eigenvalues. Systems, for which the eigenvalues of the state matrix A can be so divided, will be called <u>two-time-scale</u>. Finer subdivisions of eigenvalues into slow, fast, and very fast categories could be treated by repeating our procedure. For our initial value problems, we'll also assume <u>fast-mode-stability</u>, i.e. that the $n_2 = n - n_1$ large eigenvalues of A also have large negative real parts. We are thereby eliminating consideration of systems with large, purely imaginary

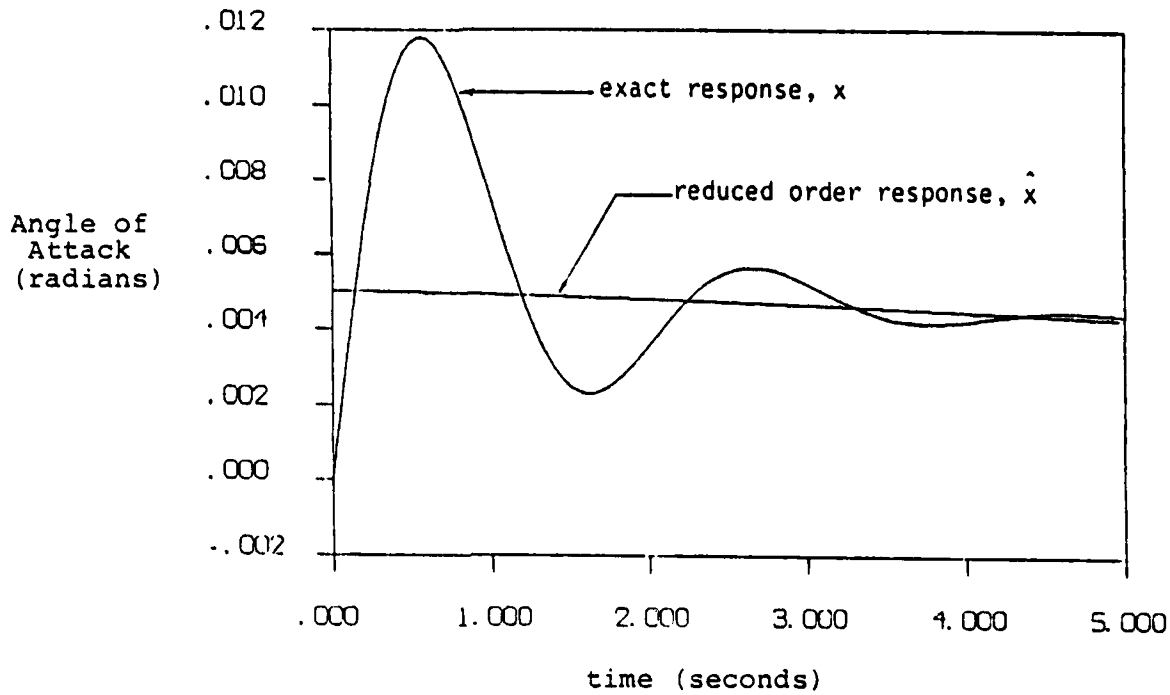

FIGURE 1. F-8 aircraft model: Angle of attack (a fast variable) vs. time.

eigenvalues. Their rapidly oscillatory solutions require techniques (cf. Miranker and Wahba (1976) or Petzold (1978)) much different from the boundary-layer method we wish to develop.

One moderate-size problem we've solved is a recently popular contest problem in the control literature (cf. Sain (1977) and De Hoff and Hall (1979)). It is a sixteenth order model for a turbofan engine. Indeed, it is one of thirty six different linearizations used to simulate the engine. The sixteen eigenvalues of A are all stable and have magnitudes ranging from 0.65 to 577. Their distribution suggests using either a third or a fifth order reduced model. We obtained very good approximations in both cases for $t > 10\varepsilon$, with ε being determined in terms of the decay rate of the smallest large eigenvalue (cf. O'Malley and Anderson (1980)).

To proceed, let us transform the constant coefficient problem

$$\dot{x} = Ax + Bu \tag{1}$$

into a new problem

$$\dot{y} = \tilde{A}y + \tilde{B}u \ , \tag{2}$$

where the fast and slow modes of the homogeneous problem are decoupled.

Specifically, let

$$y = Tx , \tag{3}$$

where

$$\tilde{A} = TAT^{-1} = \begin{pmatrix} \tilde{A}_{11} & 0 \\ 0 & \tilde{A}_{22} \end{pmatrix}$$

and

$$\tilde{B} = TB$$

for

$$T = T_1 T_2 = \begin{pmatrix} I_{n_1} & 0 \\ L & I_{n_2} \end{pmatrix} \begin{pmatrix} I_{n_1} & K \\ 0 & I_{n_2} \end{pmatrix} . \tag{4}$$

Here, $\tilde{A}_{11}$ shall have the n_1 small eigenvalues of A and $\tilde{A}_{22}$ its n_2 (relatively) large eigenvalues (with large negative real parts as well). We note that

$$T^{-1} = T_2^{-1} T_1^{-1} = \begin{pmatrix} I_{n_1} & -K \\ -L & I_{n_2} + LK \end{pmatrix} , \tag{5}$$

since, e.g., $T_1^{-1} = \begin{pmatrix} I_{n_1} & 0 \\ -L & I_{n_2} \end{pmatrix}$. Partitioning $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ compatibly,

$$T_1 A T_1^{-1} = \begin{pmatrix} \tilde{A}_{11} & A_{12} \\ 0 & \tilde{A}_{22} \end{pmatrix} \equiv \begin{pmatrix} A_{11} - A_{12}L & A_{12} \\ 0 & A_{22} + LA_{12} \end{pmatrix} \tag{6}$$

provided the $n_2 \times n_1$ matrix L satisfies the algebraic Riccati equation

$$LA_{11} - A_{22}L - LA_{12}L + A_{21} = 0 . \tag{7}$$

(The connection to invariant imbedding is signaled by the entrance of a Riccati equation.) This matrix quadratic equation will have many solutions, but only one which decouples the fast modes of the homogeneous problem, as in (6). Indeed, if the spectrum of $\tilde{A}_{11}$ coincides with the slow eigenvalues λ_{s_i}, $i = 1,\ldots,n_1$, of A, i.e. if

$$\lambda(\tilde{A}_{11}) = \left\{\lambda_{s_1},\ldots,\lambda_{s_{n_1}}\right\} , \tag{8}$$

it is easy to show that

$$L = -M_{21}M_{11}^{-1} , \tag{9}$$

where the $n \times n_1$ matrix $\begin{bmatrix} M_{11} \\ M_{21} \end{bmatrix}$ spans the n_1 dimensional slow eigenspace of A (we rearrange the rows of A, if necessary, so that M_{11} is invertible) (cf. Anderson (1978), Medanic (1979), and van Dooren (1980) (which develops numerically stable computational methods)). Rather than compute this eigenspace, we prefer to iterate in a rearranged version of the Riccati equation (7) and obtain L as the limit of iterates

$$L_{i+1} = (A_{22} + L_i A_{12})^{-1}(L_i A_{11} + A_{21}) \; . \tag{10}$$

Not unexpectedly, this procedure relates closely to Stewart (1975)'s method of finding the dominant eigenspace of A. The iteration can be shown to be robust with respect to its initialization, and its rate of convergence is proportional to the time-scale separation parameter μ, i.e. the ratio of the largest slow eigenvalue in magnitude to the smallest fast eigenvalue in magnitude.

Using the L so obtained, we will next block diagonalize the $\tilde{A}$ of (2) provided the $n_1 \times n_2$ matrix K satisfies the linear (Liapunov) equation

$$K\tilde{A}_{22} - \tilde{A}_{11}K + A_{12} = 0 \; . \tag{11}$$

Since $\tilde{A}_{11}$ and $\tilde{A}_{22}$ have no eigenvalues in common, the linear system has a unique solution (cf., e.g., Bellman (1970)) which can be found as the limit of the iteration

$$K_{j+1} = (\tilde{A}_{11}K_j - A_{12})\tilde{A}_{22}^{-1} \; . \tag{12}$$

Again, the procedure has a rate of convergence proportional to μ.

Having now determined the transforming matrix T of (4), there remains a decoupled system

$$\dot{y}_1 = \tilde{A}_{11}y_1 + \tilde{B}_1 u \tag{13}$$

and

$$\dot{y}_2 = \tilde{A}_{22}y_2 + \tilde{B}_2 u \; , \tag{14}$$

where $y = Tx = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ for $y_i = \tau_i x$ and $\tilde{B}_i = \tau_i B$ for $\tau_1 = (I_{n_1} + KL \;\; K)$ and and $\tau_2 = (L \;\; I_{n_2})$. We note that this decoupling procedure has used the the two-time-scale hypothesis, but not the hypothesis requiring fast-mode stability.

Since we've assumed that the eigenvalues of $\tilde{A}_{22}$ lie far into the left half plane, we can expect y_2 to decay rapidly to its steady-state

$$y_{2s} = -\tilde{A}_{22}^{-1}\tilde{B}_2 u \tag{15}$$

(presuming u doesn't vary rapidly). A good approximation to y away from t = 0 will then follow by simply integrating the nonstiff system of differential equations (3) of order n_1 with initial vector $y_1(0) = \tau_1 x(0)$. Returning to the original variables, we will obtain the slow-mode approximation

$$x_s(t) = T^{-1}\begin{pmatrix} y_1(t) \\ y_{2s}(t) \end{pmatrix} . \tag{16}$$

It is generally a poor approximation near t = 0 because the "boundary layer correction" $y_{2f} \equiv y_2 - y_{2s}$ is neglected. If a better approximation is needed there, one can integrate the linear system $\dot{y}_{2f} = \tilde{A}_{22}y_{2f} + \dot{y}_{2s}$ with $y_{2f}(0) = \tau_2 x(0) - y_{2s}(0)$. Because the eigenvalues of $\tilde{A}_{22}$ are large and (very) stable, we will need to use a short stepsize only on a short initial interval, since y_{2f} should rapidly decay to zero. This would fail to be true only if y_{2s} were not slowly-varying (i.e., $\dot{y}_{2s}$ were not small) for t > 0.

A more substantial problem results for initial value problems for the time-varying system

$$\dot{x} = A(t)x + B(t)u(t) \tag{17}$$

on $0 \leq t \leq T$. Assuming A(0) is two-time scale with n_1 slow modes, we can seek a transformation T, as in (4), but now with time-varying decoupling matrices L(t) and K(t) determined so that the transformed problem for

$$y = T(t)x \tag{18}$$

remains two-time-scale and fast-mode stable with the same fixed number n_1 of (relatively) small eigenvalues throughout $0 \leq t \leq T$.

We must cautiously note that eigenvalue stability does not imply stability for time-varying systems (cf., e.g., Coppel (1978)), but this is nearly true for certain singularly perturbed systems satisfying appropriate stability hypotheses (cf. the statement of Tikhonov's theorem in Wasow (1965) or Vasil'eva and Butuzov (1973)). Kreiss (1978, 1979) exhibits counterexamples, however, for systems with non-smooth coefficients.

Using our transformation (4), $L(t)$ will block-triangularize the system matrix $A(t)$ provided it now satisfies the matrix Riccati differential equation

$$\dot{L} = -LA_{11}(t) + A_{22}(t)L + LA_{12}(t)L - A_{21}(t) \ . \tag{19}$$

If $\dot{L}(0) = 0$, the transformation $T_1(0)$ will be a similarity transformation, and we can ask (as before) that the eigenvalues of $\tilde{A}_{11}(0) = A_{11}(0) - A_{12}(0)L(0)$ coincide with the n_1 slow eigenvalues of $A(0)$. The initial matrix $L(0)$ can then be obtained, as before, by iterating in the algebraic Riccati equation $\dot{L}(0) = 0$. We will then obtain $L(t)$ on $0 \leq t \leq T$ by integrating the resulting initial value problem. We would stop the integration and abandon our procedure if we encountered a finite escape time, or if the transformed problem ceased to be two-time-scale and fast-mode-stable within our t interval. (Existence criteria for the symmetric matrix Riccati equation commonly encountered in control theory are known, but analogous criteria for the non-square problem do not seem to be available.) The possibility for intermittent reinitialization should be investigated, corresponding to the reorthonormalizations of Scott and Watts (1977).

Knowing an appropriate $L(t)$, the matrix $K(t)$ will produce a block-diagonalization of the state matrix provided it satisfies the linear differential system

$$\dot{K} = \tilde{A}_{11}(t)K - K\tilde{A}_{22}(t) - A_{12}(t) \ . \tag{20}$$

Since fast-mode stability implies that the eigenvalues of $\tilde{A}_{22} = A_{22} + LA_{12}$ have large negative real parts, singular perturbation concepts (cf., e.g., O'Malley (1974)) suggest that the solution of the initial value problem for K, like that for $\varepsilon\dot{z} = z$, will blow up for $t > 0$. Instead, it is natural to integrate a terminal value problem for K and expect that an asymptotically valid approximation will result by using its "pseudo-steady state" approximation, i.e. the unique solution of the algebraic system

$$K_s(t) = (\tilde{A}_{11}(t)K_s(t) - A_{12}(t))\tilde{A}_{22}^{-1}(t) \ . \tag{21}$$

The need for nonuniform convergence at terminal time will be eliminated by picking $\dot{K}(T) = 0$, i.e. $K(T) = K_s(T)$.

Since the homogeneous variational equation for $L(t)$, $\dot{\ell} = -\tilde{A}_{11}\ell + \ell\tilde{A}_{22}$, is also "singularly perturbed", but opposite in stability to the system for K, we might also attempt to approximate $L(t)$ for $t > 0$ as a smooth solution $\tilde{L}(t)$ of the algebraic Riccati equation $\dot{\tilde{L}}(t) = 0$. This would certainly need $\tilde{L}(t)$ to be slowly-varying with the eigenvalues of $A_{22} + \tilde{L}A_{12}$ remaining large, and far into the left half-plane, compared to those of $A_{11} - A_{12}\tilde{L}$. Otherwise, we need to actually integrate the full initial value problem for $L(t)$.

Now note that the fast-mode stability of $\tilde{A}_{22}(t)$ implies that the slow-mode or pseudo-steady state approximation

$$y_{2s}(t) = -\tilde{A}_{22}^{-1}(t)\tilde{B}(t)u(t) \tag{22}$$

should nicely approximate y_2 for $t > 0$, provided y_{2s} remains slowly-varying. Thus, we are finally left with the need to integrate the non-stiff reduced-order system (13) for $y_1(t)$. By inverting our transformation, as in (16), we find a slow-mode approximate $x_s(t)$ for $t > 0$.

Note that all initial value problems for $x(t)$ would be solved in terms of an $n \times n$ fundamental matrix $X(t)$ for the homogeneous system. In the two-time scale situation, our problem splits into four separate problems for the $n_2 \times n_1$ matrix $L(t)$, for the $n_1 \times n_2$ matrix $K(t)$, for the $n_2 \times n_1$ fundamental matrix for $y_2(t)$, and the $n_1 \times n_1$ fundamental matrix for $y_1(t)$. With the fast-mode stability assumption, pseudo-steady state approximations can be used to eliminate the differential systems for K, y_2, and sometimes L. Thus, a substantial order reduction is achieved for the $t > 0$ approximation.

Substantially more detail regarding time-varying problems is contained in O'Malley and Anderson (1980). Various related problems might be treated similarly. If, for example, a two-time-scale system produced a matrix $\tilde{A}_{22}$ with only moderate-sized eigenvalues, one might consider y_1 to be approximately constant on finite T intervals, so that only a differential system for y_2 need be integrated. Likewise, if the eigenvalues of $\tilde{A}_{22}$ are large with both large negative and large positive real parts, one might seek reduced-order approximations for appropriate two-point problems. Finally, extensions of these ideas to nonlinear problems must be sought.

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This tutorial presentation introduces the two-time-scale concept and applies it to provide approximate solutions to initial-value problems for large-scale linear systems.

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